Title: Exploring Square Numbers, Exponents, Patterns, and Binary Numbers – Fun with Binary Birthday Candles

Brief Overview:

These lessons show concrete, pictorial, and abstract applications of exponents. The students use snap cubes to create concrete representations of square numbers, and then they create pictorial representations of square numbers. In Lesson 2, the students are introduced to exponents and fill out function tables of $y = x^2$ and $y = x^3$. In Lesson 3, the students create function tables for $y = 2^n$, and then are introduced to 8-digit binary numbers.

NCTM Content Standard:

Algebra

Students should be able to:

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use Mathematical models to represent and understand quantitative relationships

Grade/Level:

Grades 4 - 6

Duration/Length:

- Three Days (45 minutes each day); depending on the pace of your class, lessons 1 and 2, or 2 and 3 may be combined into the same day.
- There are extensions and additional practice opportunities that can be used throughout the year.

Student Outcomes:

Students will:

- Identify, describe, extend, and create a non-numeric pattern
- Create a one-operation function table
- Complete a one-operation function table
- Read, write, and represent whole numbers using symbols, words, and models.

Materials and Resources:

- Multiplication Facts Chart (poster size would be helpful)
- Snap cubes
- Copies of Student Resource Sheets
- Scissors
- Calculators (optional)
- Copy of Teacher Resource Sheet #1 (preferably printed in color and enlarged); this could be replaced with a poster of a birthday cake from a teachers' store, and adding pictures of lit candles to represent what is shown on Teacher Resource Sheet #1.

Development/Procedures:

Lesson 1 Concrete and Pictorial Exploration of Square Numbers

Preassessment – Review with students the concept of multiplication of square numbers. Remind students what square numbers are, if necessary. Look at the multiplication facts chart (Student Resource Sheet #1) and see where the square numbers are located. For example, look for 2 x 2 = 4, and 8 x 8 = 64. Have students share what they notice about the square numbers and that the square numbers lie along the diagonal.

Launch – Tell the students: "Now we will use our snap cubes to show how square numbers look." Have the students use snap cubes to represent squares for this set of square numbers: 1, 4, 9, and 16. Remind students that each snap cube model represents a **term** in a pattern, where "term" refers to the order in the pattern. After each term is created with cubes, refer back to the multiplication fact chart to have the students see how many squares are involved pictorially. (Make sure the students' models of square numbers are in fact square. For example, a 3 x 3 square will need 9 cubes, and should be shaped into a square with three rows and three columns.) Have the students look at the 4 terms they have created. The first term is one cube, which represents 1 x 1. The second term is four cubes, which represents 2 x 2. The third term is nine cubes, which represents 3 x 3; and the fourth term is sixteen cubes, which represents 4 x 4.

Teacher Facilitation – Have students describe what the next term in the pattern would look like. Have the students use centimeter grid paper (Student Resource Sheet #2) to draw what the 5 x 5 pattern would be.

Student Application - Have the students draw the next term in the pattern on their centimeter grid papers (6 x 6). By looking at their models of square numbers and their pictorial representations, ask the students to create an input / output table to represent their square numbers (Student Resource Sheet #3). Ask the students what the rule is for the input/output table they created ($y = x^2$). (This assumes the students have experience using input / output tables. Reteaching may be needed to show the students how to enter the term # into the input column, and the # of snap cubes in each model into the output column.)

Embedded Assessment – Observe that students have created the square number models and pictorial models correctly. Observe that the students created their input / output tables correctly.

Reteaching/Extension –

- For those who have not completely understood the lesson, use the multiplication facts chart and review the multiplication facts for square numbers. Have the students color in the square that represents 5 x 5. Notice that there are 5 rows and 5 columns and that they form a "square." If necessary, try using other shapes to depict growing patterns of squares.

- For students who do not have experience with input / output tables, create other types of growing patterns with snap cubes, and show the students how to enter the term number into the input column, and then the # of cubes in the term into the output column.

- For those who have understood the lesson and are ready to move ahead, have the students enter additional terms, such as 11, 20, 50, and 100 into their input / output tables. They can also create their own terms that can be squared.

Lesson 2 Introduction to Exponents

Preassessment – Review with students what square numbers are and how they are represented pictorially.

Launch – Say: "Mathematicians don't like to write a lot, so they came up with a shorter way to represent square numbers. Today we'll get to learn that shorter way." Have the students use snap cubes to represent a 3 x 3 square.

Teacher Facilitation – Introduce the concept of exponents by showing that square numbers are represented by the exponent "2". For example, 3 x 3 can be written as 3^2, and is called "3 squared." Point out that it is called "3 squared" because the 9 blocks form a square of 3 rows and 3 columns. Have students complete an input / output function table for $y = x^2$ using Student Resource Sheet #4.

Introduce the concept of x^3 by asking the students what they think 4^3 means. Explain that 4^3 means 4 x 4 x 4. It is called "4 cubed" or "4 to the 3rd power."

Student Application – Have students demonstrate with their snap cubes how to represent 2^3. They will create 3-dimensional cubes with height of 2, length of 2, and width of 2. Have students count how many cubes they used, and verify that $2^3 = 8$. Allow students to experiment with other cubic numbers (e.g., 3^3 and 4^3.) Have students create an input / output function table for $y = x^3$ using Student Resource Sheet #4.

Embedded Assessment – Observe that students created the 3-dimensional cube for 2^3 correctly. Observe that students have filled in input / output function tables correctly for $y = x^2$ and for $y = x^3$.

Reteaching/Extension –

- For those who have not completely understood the lesson, review the multiplication facts for square numbers and that mathematicians use the exponent "2" as a shortcut to show a number times itself. Therefore, $6^2 = 6 \times 6 = 36$. Have the students use a calculator to multiply 6 x 6, and see that they get the same answer when they use the exponent key on the calculator for 6^2. Have the students practice using the calculator to see the relationship between 7 x 7 x 7 and 7^3. Have the students continue practicing with other combinations.

- For those who have understood the lesson and are ready to move ahead, have the students create input / output function tables for $y = x^4$ and $y = x^5$. You could allow the students to use calculators to do the multiplication (e.g., 7 x 7 x 7 x 7) and then check their answers using the calculator's exponent function.

- Another extension would be to have the students work with powers of 10. Students should create an input / output table (Student Resource Sheet #5) and record the values for 10^1, 10^2, 10^3, etc. Ask the students to describe what patterns they see in their tables, and if they can predict what 10^{11} would look like.

Lesson 3 Introduction to Binary Numbers

Preassessment – Review with students what exponents mean, and what x^2 means. Have the students expand 3^4 and tell that it means 3 x 3 x 3 x 3, which equals 81. Give the students an expanded form, such as 8 x 8 x 8 x 8 x 8, and have them write it in exponent form, which is 8^5.

Launch – Show the picture of the birthday cake with some of the candles lit (Teacher Resource Sheet #1) and ask the students what they think it means. Say: "In understanding this birthday cake, we will learn about a real-world use of exponents that probably your parents don't even know!"

Teacher Facilitation – Say: "Let's see what we can do with exponents that will help us understand this birthday cake. We will look at the powers of 2." Using Student Resource Sheet #6, work as a class and have the students write the expanded notation and then fill in the input / output table for each row of the table. Explain that mathematicians decided that any number to the zero power, such as 2^0 always equals 1. Work through 2^1 through 2^7 as a class. (If desired, you could allow students to use calculators to verify the answers.) Ask the students to look for patterns within the table of numbers. They should notice that each number is the double of the previous number in the table.

Explain that the numbers in the table are called "Powers of 2." These numbers can be used in a special way to create something called a **binary number**.

Explain that computers use binary numbers to represent numbers and letters inside the computer. Hand out the binary place value mats (Student Resource Sheet #7) and the binary numbers (Student Resource Sheet #8). Tell the students to look at their Binary Place Value Mats. They are similar to a regular place value mat, but only the numbers "0" or "1" are allowed in each slot. Each slot represents a power of 2. Starting at the farthest right slot, it starts with 2^0, then 2^1, 2^2, up to 2^7 as you move left across the page.

Have the students cut out their binary number cards (Student Resource Sheet #8). Then have the students show the number 00000110 on their mats. This binary number means $2^2 + 2^1 = 4 + 2 = 6$. (See Teacher Resource Sheet #2 for more information about binary numbers.).

Student Application – Have the students show the binary number 01010101 on their mats, and then figure out what the number means as a decimal whole number. 01010101 means $2^6 + 2^4 + 2^2 + 2^0 = 64 + 16 + 4 + 1 = 85$. Allow the students to create other binary numbers on their mats and figure out what they mean in decimal. Ask students to describe any patterns they see.

Play a fun game with the students to physically represent binary numbers, called the Binary Number Chair Game. Line up 8 chairs side-by-side in the front of the room. Have 8 students sit in the chairs. Each student represents a power of 2. Starting at the farthest right chair, that student represents 2^0, then 2^1, 2^2, up to 2^7 as you move left across the row of chairs. When a student is sitting, he/she represents "0", and when the student is standing, he/she represents "1". (If desired, each student could hold up the appropriate "power of 2" card (Teacher Resource Sheet #3) to help reinforce the place value he/she represents.) Call out different decimal numbers, such as 13, and have the students stand and sit to represent the number in binary. Have the class tell which students should be standing and sitting. Ask students to describe any patterns they see.

Refer back to the birthday cake picture (Teacher Resource Sheet #1). Say: "Now let's see if we can understand what the candles on this cake mean." Ask for guesses from the students. If needed, relate the candles to the arrangement for binary numbers. The students should see that the lit candles represent 00101101, which equals $2^0 + 2^2 + 2^3 + 2^5 = 45$. "So with only 8 candles, we should be able to represent anyone's age on the cake."

Hand out Student Resource Sheet #9. Have the students color in the flames on their birthday cakes as 00001111 and then decide what age it represents. Ask a student to give the answer (which is 15).

For continued practice, give the students Student Resource Sheet #10 for homework, or have them work it in class if you have time.

Embedded Assessment – Observe the students as they work with their binary place value mats, when they play the Binary Number Chair Game, and when they work on Student Resource Sheet #9 to see if they understand the patterns and how to form binary numbers. Verify the students' understanding by reviewing Student Resource Sheet #10 after the students have completed it for homework.

Reteaching / Extension

- For those students who have not completely understood the lesson, use Teacher Resource Sheet #4 to help work through an example of converting a binary number to a decimal number. Have the student work with the Binary Number Place Value mat and continue to create binary numbers and then convert them to decimal numbers.

- For those who have understood the lesson and are ready to move ahead, have the students try to convert decimal numbers to binary numbers. For example, how would you convert 55 to a binary number?

- As another extension, have students look at the table of binary to decimal numbers shown in Teacher Resource Sheet #2. Ask students to identify patterns that they see in the table.

- Throughout the school year, as a Math warm-up activity, periodically display the Binary Birthday Cake Poster showing different arrangements of lit candles and have the students identify what age is being represented in binary by the candles.

- Computers actually use binary numbers to represent all keyboard characters. Provide copies of Teacher Resource Sheet #5, which shows the binary representation of the ASCII alphabet. Have the students write their names in ASCII binary numbers. Students could also write coded messages to each other. This could be a fun activity to use periodically throughout the school year.

Summative Assessment:

Give the students Student Resource Sheet #11 as a summative assessment for this unit. Teacher Resource Sheet #6 provides the answers to the assessment, and Teacher Resource Sheet #7 provides the rubric to use for Brief Constructed Response (BCR) questions.

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Multiplication Facts Chart

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

CENTIMETER GRID PAPER

Rule:

Input	Output
0	
1	
2	
3	
4	
5	

Rule: x^2

Input x	Output y
0	
1	
2	
3	
4	
5	
	81
10	
20	

Rule: x^3

Input x	Output y
0	
1	
2	
3	
4	
5	
10	

"Powers of 10"

Rule: 10^n

Write in expanded notation, then put the answer in the table

$10^0 = 1$

$10^1 =$

$10^2 =$

$10^3 = 10 \times 10 \times 10$

$10^4 =$

$10^5 =$

$10^6 =$

$10^7 =$

Input n	Output y
0	1
1	
2	
3	
4	
5	
6	
7	

What pattern do you see?

What do you think 10^{11} will be?

"Powers of 2"

Rule: 2^n

Write in expanded notation, then put the answer in the table

$2^0 = 1$

$2^1 =$

$2^2 =$

$2^3 = 2 \times 2 \times 2$

$2^4 =$

$2^5 =$

$2^6 =$

$2^7 =$

Input n	Output y
0	1
1	
2	
3	8
4	
5	
6	
7	

Binary Place Value Mat

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

________ + ________ + _________ + ________ + ________ + ________ + ________ + ________

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Let's Practice Making Binary Numbers

Cut out these cards to use with the Binary Place Value Mat.

1	1	1	1	0	0	0	0
1	1	1	1	0	0	0	0

2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

Binary Birthday Candles

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$

2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

Binary Birthday Candles Practice

Color in candles to represent the binary number 00010101.

Determine what age the number is in decimal.

Explain how you know you found the correct answer.

2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

Color in candles to represent the binary number 00011101.

Determine what age the number is in decimal.

Explain how you know you found the correct answer.

Name ______________________________ Date ____________________

Let's See What You Learned

1. **Which of the following is the expanded notation for 2^4?**

    a. **4 x 4**
    b. **2 x 2**
    c. **2 x 2 x 2 x 2**
    d. **2 x 4**

2. **Which of the following is the exponent form of 27?**

    e. **3 x 9**
    f. **3 x 3 x 3**
    g. **3+3+3+3+3+3+3+3+3**
    h. **3^3**

3. **Which of the following is the expanded notation for 10^5?**

    i. **10 x 10 x 10 x 10 x 10**
    j. **100,000**
    k. **5 x 10**
    l. **10 x 50**

4. Complete the input / output table:

Rule: $y = x^2$

Input x	Output y
2	
4	
5	
	64
10	

Use what you know about exponents to explain why your answer is correct for $x = 4$. Use words and numbers in your explanation.

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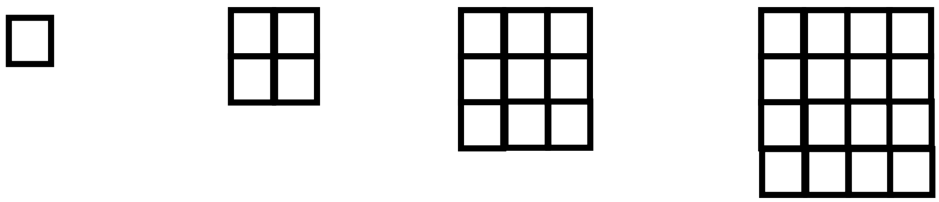
5. Create an input / output table for the following pattern:



Input x	Output y

Draw a picture of the next term in the pattern and add the term to your input / output table.

What rule do you think is represented by the pattern?

6. Look at this binary number:

00001111

<u>Part A.</u>
Convert the binary number to a decimal whole number.

<u>Part B.</u>
Use what you know about exponents and binary numbers to explain why your answer is correct. Use words and/or numbers in your explanation.

Binary and Decimal Numbers

Binary #	Decimal #
00000000	0
00000001	1
00000010	2
00000011	3
00000100	4
00000101	5
00000110	6
00000111	7
00001000	8
00001001	9
00001010	10
00001011	11
00001100	12
00001101	13
00001110	14
00001111	15
00010000	16
00010001	17
00010010	18
00010011	19
00010100	20
00010101	21
00010110	22
00010111	23
00011000	24

Binary #	Decimal #
00011001	25
00011010	26
00011011	27
00011100	28
00011101	29
00011110	30
00011111	31
00100000	32
00100001	33
00100010	34
00100011	35
00100100	36
00100101	37
00100110	38
00100111	39
00101000	40
00101001	41
00101010	42
00101011	43
00101100	44
00101101	45
00101110	46
00101111	47
00110000	48
00110001	49

Binary #	Decimal #
00110010	50
00110011	51
00110100	52
00110101	53
00110110	54
00110111	55
00111000	56
00111001	57
00111010	58
00111011	59
00111100	60
00111101	61
00111110	62
00111111	63
01000000	64
01000001	65
01000010	66
01000011	67
01000100	68
01000101	69
01000110	70
01000111	71
01001000	72
01001001	73
01001010	74
01001011	75
01001100	76

Binary #	Decimal #
01001101	77
01001110	78
01001111	79
01010000	80
01010001	81
01010010	82
01010011	83
01010100	84
01010101	85
01010110	86
01010111	87
01011000	88
01011001	89
01011010	90
01011011	91
01011100	92
01011101	93
01011110	94
01011111	95
01100000	96
01100001	97
01100010	98
01100011	99
01100100	100
01100101	101
01100110	102
01100111	103

Binary #	Decimal #
01101000	104
01101001	105
01101010	106
01101011	107
01101100	108
01101101	109
01101110	110
01101111	111
01110000	112
01110001	113
01110010	114
01110011	115
01110100	116
01110101	117
01110110	118
01110111	119
01111000	120
01111001	121
01111010	122
01111011	123
01111100	124
01111101	125
01111110	126
01111111	127
10000000	128
10000001	129
10000010	130

Binary #	Decimal #
10000011	131
10000100	132
10000101	133
10000110	134
10000111	135
10001000	136
10001001	137
10001010	138
10001011	139
10001100	140
10001101	141
10001110	142
10001111	143
10010000	144
10010001	145
10010010	146
10010011	147
10010100	148
10010101	149
10010110	150
10010111	151
10011000	152

…and the pattern continues, up to:

11111111	255

Additional information about binary numbers is available online. Try these web sites:

http://www.cs.colorado.edu/~l3d/courses/CSCI1200-96/binary.html -- provides a simple description of binary numbers

http://www.geocities.com/regia_me/index.html -- provides information about binary numbers

http://www.geocities.com/regia_me/rep_conv.htm -- provides an interactive way to see binary numbers as it lights up light bulbs to represent the binary number

http://www.geocities.com/regia_me/decToBin.htm -- converts decimal numbers to binary numbers

http://imagine.gsfc.nasa.gov/docs/teachers/lessons/slap/slap.html -- provides another idea of having students raise their arms to represent a “1” in a binary number.

http://broncgeeks.billings.k12.mt.us/vlong/dec2bin/ -- converts text letters (ASCII) to binary numbers

http://www.theskull.com/javascript/ascii-binary-list.html -- provides complete list of ASCII characters and their binary numbers

$$2^0$$

$$2^2$$

$$2^1$$

$$2^3$$

$$2^4$$

$$2^6$$

$$2^5$$

$$2^7$$

Binary Place Value Mat (example)

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
0	1	0	0	0	1	0	1

___0__ + ___64___ + ___0___ + ___0___ + ___0___ + ___4___ + ___0___ + ___1___

= 69

Let's Practice Making Binary Numbers

ASCII Text Characters and Their Binary Representations

ASCII Text Character	Binary #
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000
I	01001001
J	01001010
K	01001011
L	01001100
M	01001101
N	01001110
O	01001111
P	01010000
Q	01010001
R	01010010
S	01010011
T	01010100
U	01010101
V	01010110

ASCII Text Character	Binary #
W	01010111
X	01011000
Y	01011001
Z	01011010
?	00111111
.	00101110
,	00101100
/	00101111

Information was obtained from:
http://www.theskull.com/javascript/ascii-binary-list.html

Name ______________________________ **Date** ____________________

Let's See What You Learned *(answers)*

1. Which of the following is the expanded notation for 2^4?

- **a. 4 x 4**
- **b. 2 x 2**
- **c. $\boxed{2 \text{ x } 2 \text{ x } 2 \text{ x } 2}$**
- **d. 2 x 4**

2. Which of the following is the exponent form of 27?

- **e. 3 x 9**
- **f. 3 x 3 x 3**
- **g. 3+3+3+3+3+3+3+3+3**
- **h. $\boxed{3^3}$**

3. Which of the following is the expanded notation for 10^5?

- **i. $\boxed{10 \text{ x } 10 \text{ x } 10 \text{ x } 10 \text{ x } 10}$**
- **j. 100,000**
- **k. 5 x 10**
- **l. 10 x 50**

4. Complete the input / output table:

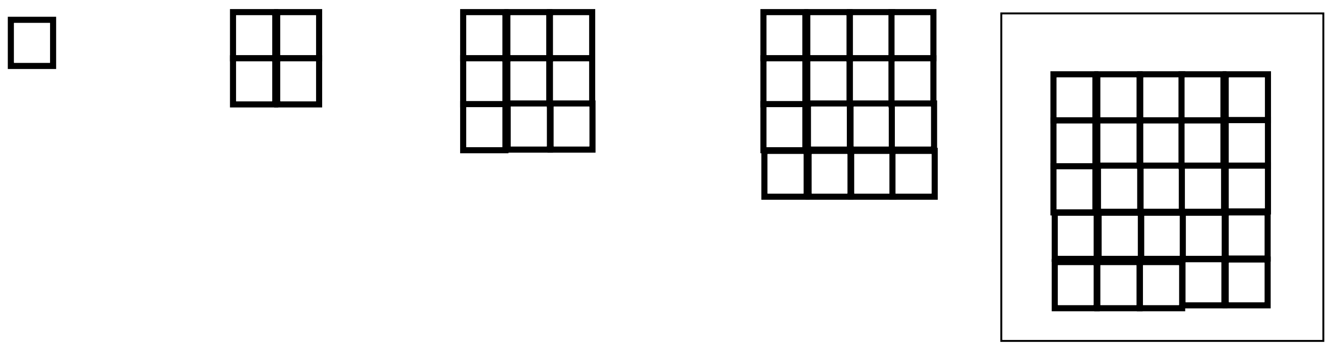
Rule: $y = x^2$

Input x	Output y
2	$\boxed{4}$
4	$\boxed{16}$
5	$\boxed{25}$
$\boxed{8}$	64
10	$\boxed{100}$

Use what you know about exponents to explain why your answer is correct for x = 4. Use words and numbers in your explanation.

I know that 4^2 means 4 x 4, which equals 16.

5. Create an input / output table for the following pattern:



Input x	Output y
1	1
2	4
3	9
4	16
5	25

Draw a picture of the next term in the pattern and add the term to your input / output table.

What rule do you think is represented by the pattern?

$y = x^2$

6. Look at this binary number:

00001111

<u>Part A.</u>
Convert the binary number to a decimal whole number.

1 + 2 + 4 + 8 = 15

<u>Part B.</u>
Use what you know about exponents and binary numbers to explain why your answer is correct. Use words and/or numbers in your explanation.

I know that binary numbers are place value holders for powers of 2. Starting at the right and moving left, they are 2^0, 2^1, 2^2, 2^3, 2^4, up to 2^7. When there is a 1 in the position, you add that value to get the decimal answer. So, for 00001111, there are 1's in the 2^0, 2^1, 2^2, and 2^3 places. So, you add 1 + 2 + 4 + 8 to get 15 as the decimal answer.

MSA Mathematics BCR Rubric
Grades 3 through 8

2 The response demonstrates a complete understanding and analysis of a problem.

- Application of a reasonable strategy in the context of the problem is indicated.
- Explanation[1] of and/or justification[2] for the mathematical process(es) used to solve a problem is clear, developed, and logical.
- Connections and/or extensions made within mathematics or outside of mathematics are clear.
- Supportive information and/or numbers are provided as appropriate. [3]

1 The response demonstrates a minimal understanding and analysis of a problem.

- Partial application of a strategy in the context of the problem is indicated.
- Explanation[1] of and/or justification[2] for the mathematical process(es) used to solve a problem is partially developed, logically flawed, or missing.
- Connections and/or extensions made within mathematics or outside of mathematics are partial or overly general, or flawed.
- Supportive information and/or numbers may or may not be provided as appropriate. [3]

0 The response is completely incorrect, irrelevant to the problem, or missing. [4]

Notes:

[1] **Explanation** refers to students' ability to communicate **how** they arrived at the solution for an item using the language of mathematics.

[2] **Justification** refers to students' ability to support the reasoning used to solve a problem, or to demonstrate **why** the solution is correct using mathematical concepts and principles.

[3] Students need to complete rubric criteria for ***explanation, justification, connections*** and/or ***explanation*** as cued for in a given problem.

[4] An exact copy or paraphrase of the problem that provides no new relevant information will receive a score of "0".